Chomsky Normal Forms

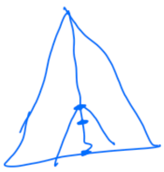
$A \to BC$

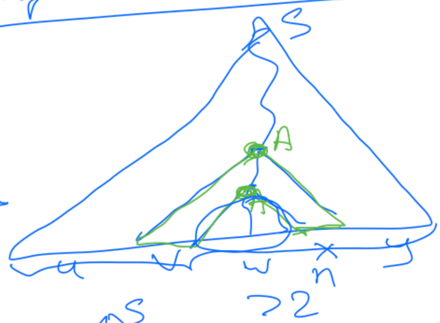
$A \to a$

For every grammar G, there an equivalent grammar G' in CNF

$$L(G') = L(G) \setminus \{\varepsilon\}$$

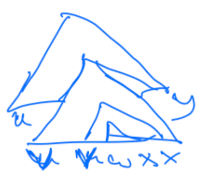
Pumping Lemma for CFL





parse tree

n: # Non terminals



Pumping lemma ~~for~~ for CFL

If L is CF, then $\exists\, n \in \mathbb{N}$

$\forall z \in L$ s.t. $|z| \geq n$

$\exists$ words $u, v, w, x, y \in \Sigma^*$
with $v \neq \varepsilon$ or $x \neq \varepsilon$

s.t. $z = uvwxy$ $\quad |vwx| \leq n$

s.t.
$uv^iwx^iy \in L \quad \forall i \geq 0$

If L is CF then it satisfies pumping lemma

$a^m b^m \mid m \geq 0$

$\exists\, n = 2 \quad \forall z$: . . . .

eg. $a^n b^n$

$ab \in L \quad |ab| \geq 2$

$u, w, y = \varepsilon$

$v = a \quad x = b$

$v^i x^i = a^i b^i \in L \quad \forall i \geq 0$

If L doesnot satisfy pumping lemma then L is not CFL

example
$L = \{a^m b^m c^m \mid m \geq 0\}$

$\forall n \in \mathbb{N}$

$\exists z$

let $z = a^n b^n c^n$

$\forall u, v, w, x, y$

s.t. $z = uvwxy$, $v \neq \varepsilon$ or $x \neq \varepsilon$

aaa.... bb---- c----

<u>Cases</u>: $v \neq \varepsilon$, $x = \varepsilon$

$a^m b^m c^m$ not CFL $\square$

<u>Greibach Normal Form</u>

$A \to a B_1 B_2 \cdots B_k$

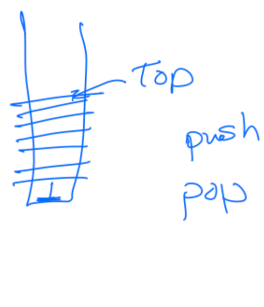
$\Sigma \times N^*$

Pushdown Automata

<u>FSA</u> + stack

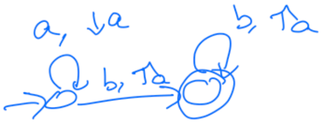
Stack Alphabet: Γ

Γ is finite.



$a^n b^m \mid n \geq m$

$\Sigma = \{a, b\}$



$a^n b^m \mid n \geq m$

'$\perp$' : bottom of the stack

$(Q, \Sigma, \Gamma, q_0, \perp, F, \delta)$

Q – finite set of states

Σ – finite input alphabet

Γ – finite stack alphabet

$q_0 \in Q$ – initial state

$\perp \in \Gamma$ – bottom stack symbol

$F \subseteq Q$ – accepting states

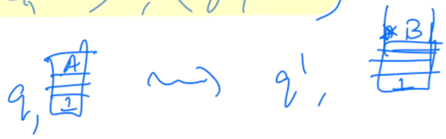
$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$

$\delta \subseteq (Q \times \Sigma_\varepsilon \times \Gamma) \times (Q \times \Gamma^*)$ – finite set of transition.

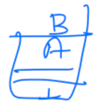
$(q, a, A), (q', w)$

$((q, a, A), (q', \varepsilon)) \in \delta$ pop A from top of stack

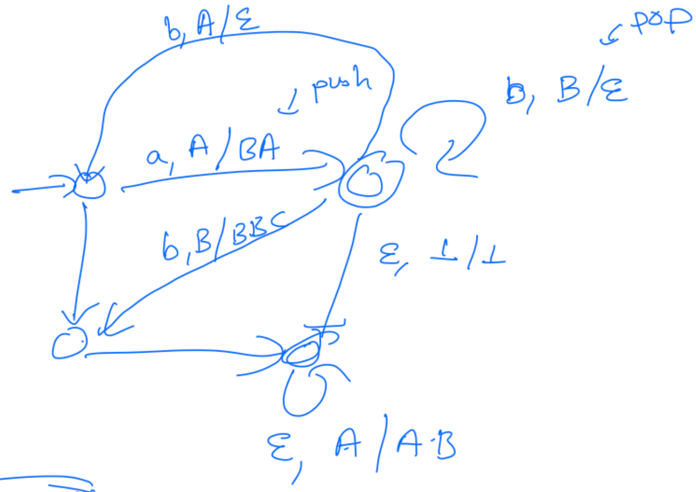
$(q, a, A), (q', B)$



$(q, a, A), (q', A)$



$(q, a, A), (q', BA)$



$Q \times \Gamma^*$

$(q, ABA) \xrightarrow{a} (q', BA)$

$(q_0, \perp)$ $(q_F, \_)$

ID: Instantaneous description

(q, w, W)

Configuration

$(q, aw, AW) \xrightarrow{a} (q', w, W)$

if $(q, a\, A)(q', \varepsilon) \in \delta$

$(q_0, w, \perp) \xrightarrow{*} (q_F, \varepsilon, W)$

"palindromes."

"well bracketed words"